MST Problem: Kruskal's Algorithm

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ALGORITHM: Find-MST-Kruskal
  INPUT: Simple, Undirected, Weighted, Connected G = (V, E)
  OUTPUT: A minimum spanning tree T of G
PROCEDURE:
  for v ∈ V: C(v) := {v} -- build |V| elementary clusters
  Initialize a priority queue Q containing E -- keyed by weights
  T := ∅
  while |T| ≠ n − 1:
    (u, v) := Q.removeMin()
    let C(u) be the cluster containing u
    let C(v) be the cluster containing v
    if C(u) ≠ C(v) then
      T := T ∪ {(u, v)}
      Merge C(u) and C(v) into one cluster
```

MST Problem: Partition, Cluster, Cut

Iteration	Min Edge	Processing	Resulting Partition	T: MST Under Construnction
Init.	—	—	$\{\{A\},\{B\},\{C\},\{D\},\{E\},\{F\},\{G\},\{H\}\}$	$\varnothing$
1	$w(A,B)=1$	$\because C(A)\neq C(B)\therefore$ Tree Edge	$\{\{A,B\},\{C\},\{D\},\{E\},\{F\},\{G\},\{H\}\}$	$\{(A,B)\}$
2	$w(B,C)=2$	$\because C(B)\neq C(C)\therefore$ Tree Edge	$\{\{A,B,C\},\{D\},\{E\},\{F\},\{G\},\{H\}\}$	$\{(A,B),(B,C)\}$
3	$w(A,D)=3$	$\because C(A)\neq C(D)\therefore$ Tree Edge	$\{\{A,B,C,D\},\{E\},\{F\},\{G\},\{H\}\}$	$\{(A,B),(B,C),(A,D)\}$
4	$w(C,D)=3$	$\because C(C)=C(D)\therefore$ Internal Edge	No Change	
5	$w(E,F)=4$	$\because C(E)\neq C(F)\therefore$ Tree Edge	$\{\{A,B,C,D\},\{E,F\},\{G\},\{H\}\}$	$\{(A,B),(B,C),(A,D),(E,F)\}$
6	$w(D,E)=5$	$\because C(D)\neq C(E)\therefore$ Tree Edge	$\{\{A,B,C,D,E,F\},\{G\},\{H\}\}$	$\{(A,B),(B,C),(A,D),(E,F),(D,E)\}$
7	$w(C,F)=6$	$\because C(C)=C(F)\therefore$ Internal Edge	No Change	
8	$w(F,G)=7$	$\because C(F)\neq C(G)\therefore$ Tree Edge	$\{\{A,B,C,D,E,F,G\},\{H\}\}$	$\{(A,B),(B,C),(A,D),(E,F),(D,E),(F,G)\}$
9	$w(E,H)=8$	$\because C(E)\neq C(H)\therefore$ Tree Edge	$\{\{A,B,C,D,E,F,G,H\}\}$	$\{(A,B),(B,C),(A,D),(E,F),(D,E),(F,G),(E,H)\}$

MST Problem: Cut Property

MST Problem: Cut Property in Kruskal's Algorithm

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Iteration	Min Edge	Processing	Resulting Partition	T: MST Under Construnction
Init.	—	—	$\{\{A\},\{B\},\{C\},\{D\},\{E\},\{F\},\{G\},\{H\}\}$	$\varnothing$
1	$w(A,B)=1$	$\because C(A)\neq C(B) \therefore$ Tree Edge	$\{\{A,B\},\{C\},\{D\},\{E\},\{F\},\{G\},\{H\}\}$	$\{(A,B)\}$
2	$w(B,C)=2$	$\because C(B)\neq C(C) \therefore$ Tree Edge	$\{\{A,B,C\},\{D\},\{E\},\{F\},\{G\},\{H\}\}$	$\{(A,B),(B,C)\}$
3	$w(A,D)=3$	$\because C(A)\neq C(D) \therefore$ Tree Edge	$\{\{A,B,C,D\},\{E\},\{F\},\{G\},\{H\}\}$	$\{(A,B),(B,C),(A,D)\}$
4	$w(C,D)=3$	$\because C(C)=C(D) \therefore$ Internal Edge	No Change	
5	$w(E,F)=4$	$\because C(E)\neq C(F) \therefore$ Tree Edge	$\{\{A,B,C,D\},\{E,F\},\{G\},\{H\}\}$	$\{(A,B),(B,C),(A,D),(E,F)\}$
6	$w(D,E)=5$	$\because C(D)\neq C(E) \therefore$ Tree Edge	$\{\{A,B,C,D,E,F\},\{G\},\{H\}\}$	$\{(A,B),(B,C),(A,D),(E,F),(D,E)\}$
7	$w(C,F)=6$	$\because C(C)=C(F) \therefore$ Internal Edge	No Change	
8	$w(F,G)=7$	$\because C(F)\neq C(G) \therefore$ Tree Edge	$\{\{A,B,C,D,E,F,G\},\{H\}\}$	$\{(A,B),(B,C),(A,D),(E,F),(D,E),(F,G)\}$
9	$w(E,H)=8$	$\because C(E)\neq C(H) \therefore$ Tree Edge	$\{\{A,B,C,D,E,F,G,H\}\}$	$\{(A,B),(B,C),(A,D),(E,F),(D,E),(F,G),(E,H)\}$