## MST Problem: Kruskal's Algorithm

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ALGORITHM: Find-MST-Kruskal
 INPUT: Simple, Undirected, Weighted, Connected G = (V, E)
 OUTPUT: A minimum spanning tree T of G
PROCEDURE:
 for v \in V: C(v) := \{v\} -- build |V| elementary clusters
 Initialize a priority queue Q containing E -- keyed by weights
 T := \emptyset
 while |T| \neq n-1:
   (u, v) := Q.removeMin()
   let C(u) be the cluster containing u
   let C(v) be the cluster containing v
   if C(u) \neq C(v) then
     T := T \cup \{(u, v)\}
    Merge C(u) and C(v) into one cluster
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## MST Problem: Partition, Cluster, Cut

ITERATION	MIN EDGE	Processing	RESULTING PARTITION	T: MST UNDER CONSTRUNCTION
TIERATION	WIIN EDGE	FROCESSING	RESULTING FARTITION	7. MST UNDER CONSTRUNCTION
Init.	_	_	$ \begin{cases} \{A\}, \{B\}, \{C\}, \{D\}, \\ \{E\}, \{F\}, \{G\}, \{H\} \end{cases} $	Ø
1	w(A,B)=1	$C(A) \neq C(B)$ $Tree$ Edge	$ \begin{cases} \{A,B\}, \{C\}, \{D\}, \\ \{E\}, \{F\}, \{G\}, \{H\} \end{cases} $	{ (A, B) }
2	w(B,C)=2	$C(B) \neq C(C)$ $\therefore$ Tree Edge	$ \left\{  \begin{cases}     \{A, B, C\}, \{D\}, \\     \{E\}, \{F\}, \{G\}, \{H\}     \end{cases} \right\} $	$\{(A,B),(B,C)\}$
3	w(A,D)=3	$C(A) \neq C(D)$ $Tree$ Edge	$ \left\{  \begin{cases}     \{A, B, C, D\}, \\     \{E\}, \{F\}, \{G\}, \{H\}     \end{cases} \right\} $	$\left\{ (A,B),(B,C),(A,D) \right\}$
4	w(C,D)=3	C(C) = C(D) : Internal Edge	No Change	
5	w(E,F) = 4	$C(E) \neq C(F)$ $Tree$ Edge	$\left\{ \begin{array}{l} \{A,B,C,D\},\\ \{E,F\},\{G\},\{H\} \end{array} \right\}$	$\left\{ (A,B),(B,C),(A,D),(E,F) \right\}$
6	w(D,E)=5	$C(D) \neq C(E)$ $Tree$ Edge	$ \left\{ \begin{array}{l} \{A,B,C,D,E,F\}, \\ \{G\},\{H\} \end{array} \right\} $	$\left\{ \begin{array}{l} (A,B),(B,C),(A,D),(E,F),\\ (D,E) \end{array} \right\}$
7	w(C,F) = 6	C(C) = C(F) : Internal Edge	No Change	
8	w(F,G) = 7	$C(F) \neq C(G)$ $\therefore$ Tree Edge	$ \left\{  \begin{cases}     \{A,B,C,D,E,F,G\}, \\     \{H\}     \end{cases} \right\} $	$\left\{ \begin{array}{l} (A,B),(B,C),(A,D),(E,F),\\ (D,E),(F,G) \end{array} \right\}$
9	w(E,H) = 8	$C(E) \neq C(H)$ $\therefore$ Tree Edge	$\left\{ \left. \{A,B,C,D,E,F,G,H\} \right. \right\}$	$ \left\{ \begin{array}{l} (A,B), (B,C), (A,D), (E,F), \\ (D,E), (F,G), (E,H) \end{array} \right\} $

## MST Problem: Cut Property

## MST Problem: Cut Property in Kruskal's Algorithm

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ALGORITHM: Find-MST-Kruskal
          INPUT: Simple, Undirected, Weighted, Connected G = (V, E)
          OUTPUT: A minimum spanning tree T of G
       PROCEDURE .
          for v \in V: C(v) := \{v\} -- build |V| elementary clusters
          Initialize a priority queue Q containing E -- keyed by weights
          T := \emptyset
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          while |T| \neq n-1:
            (u, v) := Q.removeMin()
            let C(u) be the cluster containing u
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            let C(v) be the cluster containing v
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            if C(u) \neq C(v) then
                                                                                        MIN EDGE
                                                                                                          PROCESSING
                                                                                                                                RESULTING PARTITION
                                                                                                                                                      T: MST UNDER CONSTRUNCTION
                                                                             ITERATION
               T := T \cup \{(u, v)\}
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                                                                                                                                 \{A\}, \{B\}, \{C\}, \{D\},
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              Merge C(u) and C(v) into one cluster
                                                                                                                                 \{E\}, \{F\}, \{G\}, \{H\}
                                                                                                                                 \{A, B\}, \{C\}, \{D\},
                                                                                                                                                               \{(A,B)\}
                                                                                                    C(A) \neq C(B) Tree Edge
                                                                                        w(A, B) = 1
                                                                                                                                 \{E\}, \{F\}, \{G\}, \{H\}
                                                                                                                                 \{A, B, C\}, \{D\},
                                                                                        w(B,C) = 2 \therefore C(B) \neq C(C) \therefore Tree Edge
                                                                                                                                                            \{(A,B),(B,C)\}
                                                                                                                                 \{E\}, \{F\}, \{G\}, \{H\}
                                                                                                                                 \{A, B, C, D\},\
                                                                                                                                                          \{ (A,B), (B,C), (A,D) \}
                                                                                        w(A, D) = 3 : C(A) \neq C(D) : Tree Edge
                                                                                                                                 \{E\}, \{F\}, \{G\}, \{H\}
                                                                                        w(C, D) = 3 : C(C) = C(D) : Internal Edge
                                                                                                                                                   No Change
                                                                                                                                  \{A, B, C, D\},\
                                                                                                                                                       \{(A,B),(B,C),(A,D),(E,F)\}
                                                                                        w(E,F) = 4 \therefore C(E) \neq C(F) \therefore Tree Edge
                                                                                                                                  \{E,F\},\{G\},\{H\}
                                                                                                                                  \{A, B, C, D, E, F\},\
                                                                                                                                                        (A, B), (B, C), (A, D), (E, F),
                                                                                        w(D, E) = 5 : C(D) \neq C(E) : Tree Edge
                                                                                                                                  \{G\}, \{H\}
                                                                                                                                                        (D, E)
                                                                                        w(C, F) = 6 \therefore C(C) = C(F) \therefore Internal Edge
                                                                                                                                                   No Change
                                                                                                                                 \{A, B, C, D, E, F, G\},
                                                                                                                                                        (A, B), (B, C), (A, D), (E, F),
                                                                                        w(F,G) = 7 \therefore C(F) \neq C(G) \therefore Tree Edge
                                                                                                                                \{H\}
                                                                                                                                                        (D,E),(F,G)
                                                                                                                                                       (A,B),(B,C),(A,D),(E,F),
                                                                                        w(E, H) = 8 : C(E) \neq C(H) : Tree Edge
                                                                                                                               \{A,B,C,D,E,F,G,H\}
                                                                                                                                                       (D, E), (F, G), (E, H)
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